VP160 Recitation Class

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FoR and Newton's Law

Simple Harmonic Oscillation

Damping Oscillation

Driven Oscillation

Force and Inertial Frame of Reference

Force

Interaction between two objects or an object and the environment.

Inertial Frame of Reference

In a inertial frame of reference, if the net force on a particle is zero, then its acceleration is zero.

Newton’s Law

1. A particle acted upon by zero net force moves with constant velocity.
2. In an inertial frame of reference, acceleration of a particle is directly proportional to the net force acting upon it, and inversely proportional to its mass.
3. The mutual forces of action and reaction between two bodies are equal in magnitude and opposite in direction.

Motion with Air/Fluid Drag

Consider a falling partical with linear drag and initial conditions $y(0) = 0, v_y(0) = 0$.

$$v_y(t) = \frac{mg}{k}(1 - e^{-\frac{k}{m}t})$$

$$y(t) = \frac{mg}{k}(t + \frac{m}{k}(e^{-\frac{k}{m}t} - 1))$$

How to derive the above formulas?

Simple Harmonic Oscillator

$$\ddot{x} + \frac{k}{m}x = 0$$
$$x = Acos(\omega_0 t) + Bsin(\omega_0 t)$$
$$F = -kx$$

Question:

1. What if the equation is $\ddot{x} + \frac{k}{m}x = C$?
2. What if the equation is $\ddot{x} - \frac{k}{m}x = 0$?

General Steps - Dynamic methods

1. Apply Newton's laws to the system.
2. Write dynamic equations and solve them.
3. Find a general expression of $F = -kx$.
4. Then $\omega = \sqrt{\frac{k}{m}}$.

General Steps - Energy methods

1. Write down the total energy of the system.
2. Find a general expression of $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 + C$.
3. Then $\omega = \sqrt{\frac{k}{m}}$.

General Steps - Geometry methods

1. Apply geometry to the system.
2. Find equivalent pendulum length l.
3. Then $\omega = \sqrt{\frac{g}{l}}$.

General Steps - Differential methods

1. Write down the total energy of the system.
2. $\frac{dE}{dt} = 0$, obtain $\ddot{x} + \omega^2 x = 0$

Question

Using the four methods above to find the period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

Exercise Time

Damping Oscillation

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

1. Overdamped: $b^2 > 4km$
   $x(t) = C_1 e^{-(\frac{b}{2m}+\sqrt{\frac{b^2}{4m^2}-\omega_0^2})t} + C_2 e^{-(\frac{b}{2m}-\sqrt{\frac{b^2}{4m^2}-\omega_0^2})t}$
2. Critically damped: $b^2 = 4km$
   $x(t) = C_1 e^{-\frac{b}{2m}t} + C_2 t e^{-\frac{b}{2m}t}$
3. Underdamped: $b^2 < 4km$
   $x(t) = e^{-\frac{b}{2m}t}[Acos(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}t) + Bsin(\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}t)]$

Driven Oscillation

Equations

$$\ddot{x}(t) + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m} cos\omega_{dr} t$$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{dr}^2)^2 + (\frac{b\omega_{dr}}{m})^2}}$$

$$tan\phi = \frac{b\omega_{dr}}{m(\omega_{dr}^2 - \omega_0^2)}$$

$$\omega_{res} = \sqrt{\omega_0^2 - \frac{b^2}{2m^2}}$$

Reminders

1. The resonance frequency is lower than the natural frequency if we have drag.
2. The response of the system is not in phase with the drive.